Time-Frequency Analysis of Radar Signals

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1. INTRODUCTION

It has been well understood that a given signal can be represented in an infinite number of different ways. Of course, different signal representations can be used for different applications. Despite the fact that the number of ways describing a given signal are countless, the most "popular", important and fundamental variables are: time and frequency. The time domain indicates how a signal's amplitude changes over time and the frequency domain indicates how often these changes take place.

The key for the description of a signal was to find a form which would unite the variables above. The tool that matched time and frequency was the "Fourier Transform". Since its introduction in the nineteenth century, the Fourier Transform has become one of the most widely used signal-analysis tools across many disciplines of science and engineering. The fundamental idea behind Fourier Transform is the decomposition of a signal as the sum of weighted sinusoidal functions of different frequencies. The projection of the values of these sinusoidal functions (each of which is a function with a unique frequency) form the Fourier Transform of the original signal.

Despite their simple interpretation of pure frequencies, the Fourier transform is not always the best tool to analyze "real life signals". These are usually of finite, perhaps even relatively short duration, and they have frequency contents that change over time. The most common examples of such signals are biomedical, musical and seismic signals. Especially seismic signals are not like sinusoidal functions, extending from negative infinity to positive infinity in time. For such kind of applications, the sinusoidal functions are not good models.

Joint Time Frequency Transforms were developed for the purpose of characterizing the time-varying frequency content of a signal. Many transforms were developed and used at different applications. The developed transforms are divided into two classes:

- Linear Time Frequency Transforms, and
- Quadratic (Bilinear) Transforms.

At the first class belong a lot of transforms. The most-known are Short Time Fourier Transform (STFT), Continuous Wavelet Transform (CWT) and Adaptive Time-Frequency Representation. At the second class belongs the Wigner-Ville Distribution (WVD). Scientific research, in the last decade, has focused on Short Time Fourier Transform modifications and especially on Wigner-Ville Distribution. [1]

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2. FOURIER TRANSFORM

The oldest method for signal processing is Fourier Transform. The Fourier Transform of a signal s(t) is defined as:

$$S(\omega)=\int_{-\infty}^{+\infty} s(t)\exp(-j\omega t)dt$$

where ω=2πf is the angular frequency. The original function can be constructed from the processing values by the process of:

$$s(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} S(\omega)\exp(j\omega t)d\omega$$

The above function is known as Inverse Fourier Transform.

Uncertainty Principle

A well-known principle of the Fourier Transform is the uncertainty principle, or Heisenberg inequality. According to this property, the time duration Δ_t of a signal s(t) and the frequency bandwidth Δ_ω of the Fourier Transform S(ω) are related by:

$$\Delta_t \Delta_\omega \geq \frac{1}{2}$$

The definitions of the above functions are:

Time Duration	$\Delta_t = \left[\frac{\int_{-\infty}^{\infty}(t-\mu_t)^2\lvert s(t)\rvert^2 dt}{\int_{-\infty}^{\infty}\lvert s(t)\rvert^2 dt}\right]^{\frac{1}{2}}$
Frequency Duration	$\Delta_\omega = \left[\frac{\int_{-\infty}^{\infty}(\omega-\mu_\omega)^2\lvert S(\omega)\rvert^2 d\omega}{\int_{-\infty}^{\infty}\lvert S(\omega)\rvert^2 d\omega}\right]^{\frac{1}{2}}$
Mean Time	$\Delta_t = \frac{\int_{-\infty}^{\infty} t\lvert s(t)\rvert^2 dt}{\int_{-\infty}^{\infty}\lvert S(t)\rvert^2 dt}$

Mean Frequency	$\Delta_\omega = \dfrac{\int_{-\infty}^{\infty} \omega \lvert S(\omega) \rvert^2 d\omega}{\int_{-\infty}^{\infty} \lvert S(\omega) \rvert^2 d\omega}$

As can be assumed from above, time resolution and frequency resolution cannot be arbitrarily small. Specifically, smaller time duration of s(t) means greater frequency bandwidth of S(ω) and vice versa [2]. The equality of the uncertainty principle is valid in case of Gaussian signals.

3. SHORT-TIME FOURIER TRANSFORM

One of the best-known time-frequency representations of a time signal dates back to Gabor and is known as "Short Time Fourier Transform - STFT". STFT is based on the Fourier Transform and its basic idea is a moving window Fourier Transform [3]. The window is moving over the time domain and the examination of the frequency content of the signal generates a 2-D time-frequency distribution called "spectrogram". The STFT of a signal s(t) is defined as:

$$STFT(t,\omega)=\int s(t')w(t'-t)\exp(-j\omega t')dt'$$

where w(t'-t) is the moving window. The only difference from Fourier Transform is the presence of the window function. The definition of STFT can be also expressed in the frequency domain as:

$$STFT(t,\omega)=\frac{1}{2\pi}\exp(-j\omega t)\int S(\omega')W(\omega'-\omega)\exp(-j\omega' t)d\omega'$$

where W(ω'-ω) is the Fourier Transform of the moving window. At the first definition the window is moving at the time domain while at the second definition is moving at the frequency domain. Many window functions are used, each of them at different application. Some of them are known as: Hamming, Hanning, Kaiser-Bessel and Gaussian windows.

STFT has two major advantages. First of all, according to its definition, STFT is simple enough, as it is equal with the computation of multiple Fourier Transforms. The second advantage is the absence of cross terms. As the width of the time window is getting smaller, the cross terms are limited, in contrast to bilinear transforms (i.e. Wigner-Ville distribution).

The major disadvantage of STFT is that during processing, the results are not good in both time and frequency domain. The processed signal can be either analyzed with good time resolution or frequency resolution [2]. This disadvantage is a characteristic of the Fourier Transform which is transferred to the STFT. According to the uncertainty principle, the functions Δ_t and Δ_ω are proporately inverse and their value is equal to the width of the moving window and the frequency bandwidth. In addition, any component of the signal, whose time duration is smaller than the time duration of the window is "disappeared" after the transform of the signal.

To overcome these limitations of the STFT, in order to obtain a multi-resolution analysis, wavelet transforms are used.

CONTINUOUS WAVELET TRANSFORM

General

The spectrogram generated by STFT is limited in resolution by the extent of the sliding window function. Contrary to the fixed resolution of the STFT, the Continuous Wavelet Transform – CWT is a time-frequency representation capable of achieving variable resolution in one domain (time or frequency) and multiresolution in the other domain [3].

The CWT was generated by a scientific group consisted of P. Goupillaud, A. Grossman and J. Morlet at 1984 [4]. The basic idea of the CWT is the "wavelet theory" which is the reason of the generation of many categories of signal processing. The most-known applications are the multi-resolution signal processing, the sub band coding and the wavelet series expansion.

The research of I. Daubechies was a great assistance at the exponential prevalence of wavelet transforms at the telecommunication and the signal-processing domain, as he generated its mathematical theory [5].

As the Wavelet Transform is an advancement of the Fourier Transform, it is also divided in two classes:

- Continuous Wavelet Transform, and
- Discrete Wavelet Transform

A main characteristic of the wavelet transforms is a new term called "scale" which displays the frequency, the basic variable of Fourier Transforms. According to that, in wavelet transforms is introduced the "time-scale representation".

Wavelet Theory

The "scale" is the basic variable of wavelet transform for non-stationary signals. Small values of scaling are proporal to high values of frequency and vice-versa. At wavelet theory, instead of the variable of time is used the variable of "shift".

A wavelet is just a waveform of finite time duration. The basic difference of the sinusoidal function is that wavelet functions are asymmetrical and irregular.

The basic idea behind wavelet analysis is the decomposition of the signal into varying wavelet function, called "mother wavelet". A fundamental condition of a mother wavelet is the "admissibility condition" [6]. The satisfaction of the above condition is necessary for the existence of inverse transform. The mother wavelet has to satisfy the next function:

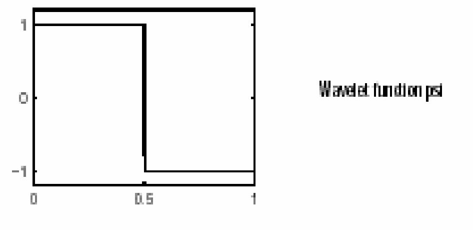
$$\int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega < \infty$$

where $\Psi(\omega)$ is the Fourier transform of the mother wavelet. In addition to that, it is known that the admissibility condition is equivalent to the nullity of the mother wavelet's DC component.

Many mother-wavelets are used for different applications. The most known are Morlet, Daubechies and Haar wavelets. Of great importance are also the coiflet, symlet, Mexican Hat and biorthogonal wavelets.

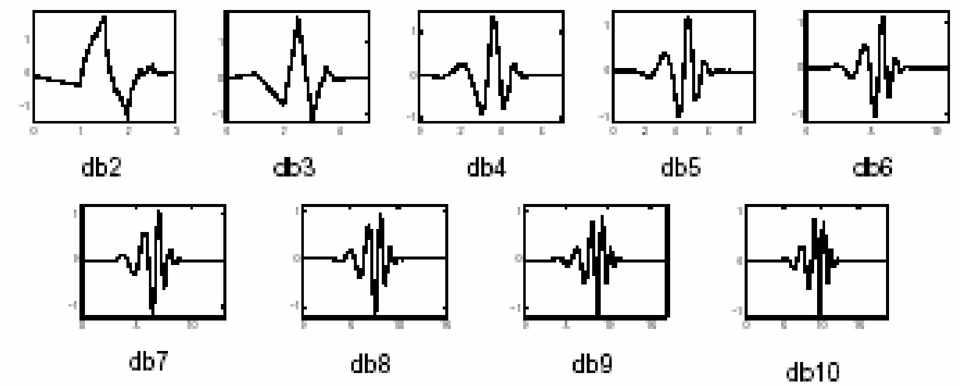
Haar

Any discussion concerning wavelets starts with Haar wavelet, the first and simplest. Haar wavelet is discontinuous, and resembles to a step function. It represents the same wavelet as Daubechies db1.



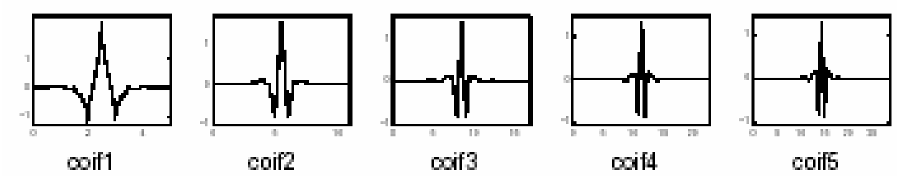
Daubechies

Ingrid Daubechies, one of the most significant scientists in the world of wavelet research, invented what are called compactly supported orthonormal wavelets — thus making discrete wavelet analysis practicable. The names of the Daubechies family wavelets are written dbN, where N is the order, and db the "surname" of the wavelet. The db1 wavelet, as mentioned above, is the same as Haar wavelet. Here are the wavelet functions psi of the next nine members of the family:



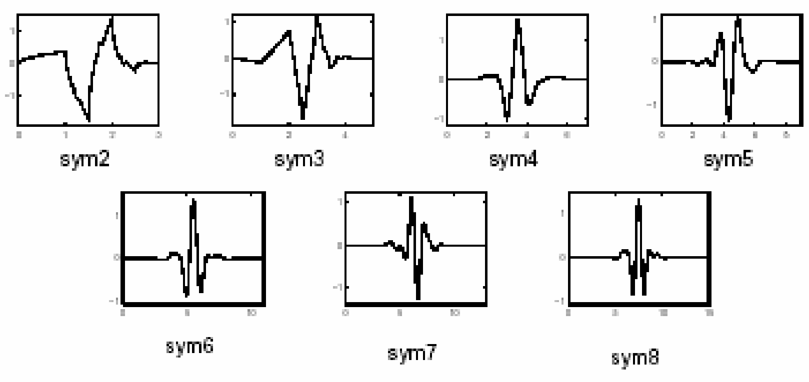
Coiflets

Built by I. Daubechies at the request of R. Coifman. The wavelet function has $2N$ moments equal to 0 and the scaling function has $2N$-1 moments equal to 0. The two functions have a support of length $6N$-1.



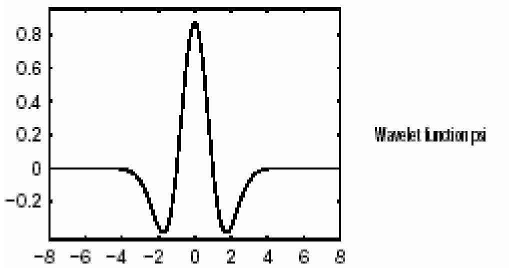
Symlets

The symlets are nearly symmetrical wavelets proposed by Daubechies as modifications to the db family. The properties of the two wavelet families are similar. Here are the wavelet functions psi.



Mexican Hat

This wavelet has no scaling function and is derived from a function that is proportional to the second derivative function of the Gaussian probability density function.



Mathematical Definition

The CWT of a signal s(t) can be defined as:

$$\mathrm{CWT}(t,\omega)=\left(\frac{\omega}{\omega_0}\right)^{1/2}\int s(t')\psi^*\left(\frac{\omega}{\omega_0}(t'-t)\right)dt'$$

where $\psi(*)$ is the mother wavelet [3]. The scale parameter is expressed as the ratio $\frac{\omega}{\omega_0}$. The 2-D result is the scalogram of the transform. If the mother wavelet is centered at time zero and oscillates at frequency ω_0 , the definition of the Continuous Wavelet Transform is the decomposition of the signal s(t') into shifted and dilated wavelets $\psi[(\omega/\omega_0)(t'-t)]$.

The basic advantage of the CWT over the STFT is the multi-resolution property. According to the above mathematical definition, the wavelet $\psi(*)$ has two variables: time and frequency. By shifting or dilating the wavelet $\psi(*)$ at a fixed parameter t or ω, signal processing is accomplished according to time or scale parameter.

The wavelet transform can also be expressed as:

$$\mathrm{CWT}(t,\omega)=\frac{\left(\omega_0/\omega\right)^{1/2}}{2\pi}\int S(\omega')\Psi^*\left(\frac{\omega_0}{\omega}\omega'\right)\exp(j\omega' t)d\omega'$$

where $\Psi(\omega')$ is the Fourier transform of $\psi(t')$. The above equation is also defined as the Fourier transform of $S(\omega')\Psi^*\left(\frac{\omega_0}{\omega}\omega'\right)$.

A comparison between STFT and CWT shows that $\Psi^*(\omega)$ is similar to the frequency window $W(\omega)$. Of course, the admissibility condition has to be satisfied, which means $\Psi(0)=0$.

In CWT the uncertainty condition is satisfied but in a different way than the STFT. At CWT the time analysis is better at high frequencies and the scale analysis is better at lower frequencies.

The basic difference between the Fourier Transform and the STFT and CWT is that, in the second case, functions based at finite extent are used, and that is the reason they are used in many applications.

5. COMPARISON BETWEEN STFT AND CWT USING MATLAB

As it has already been mentioned, Time-Frequency Transforms are ideal for a great variety of signal processing applications. In our research, the goal is to use Space Time Frequency and Continuous Wavelet Transforms, in order to extract the features of a moving target (i.e. aircraft) using the information provided by radar systems. In particular, we focus on the process of reconstructing images of radar targets from recorded data, using time-frequency transformation.

The first step of the research, described in this section, is the thorough examination of all these different categories of window functions, in STFT, and mother wavelets, in CWT. In order to examine and find out the similarities and differences between window functions or between mother wavelets, as well as between STFT and CWT, we used simple functions as examples.

In the following example a simple sine function, consisted of four sins of different frequency, is presented. The first diagram shows our function, whereas the next diagrams give a detailed comparison between all the parameters concerning window functions and mother wavelets.

$$s(t) = \begin{cases} \sin(2\pi f_1 t), & 0 < t \leq 250\text{sec} \\ \sin(2\pi f_2 t), & 250 < t \leq 500\text{sec} \\ \sin(2\pi f_3 t), & 500 < t \leq 750\text{sec} \\ \sin(2\pi f_4 t), & 750 < t \leq 1000\text{sec} \end{cases}$$

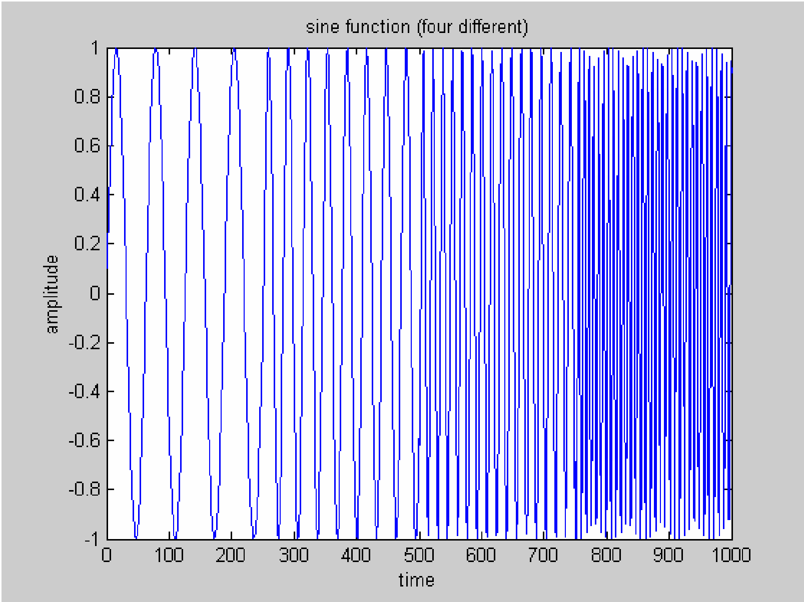


Fig.1. Signal under processing

In the following step, we calculated the STFT (spectrogram) of our signal using the Gaussian pulse as window function. The Gaussian wavelet is described as:

$$w(t) = \frac{1}{\pi^{1/4}\sqrt{\sigma}} e^{-\frac{t^2}{2\sigma^2}}$$

The parameter σ is the one of great importance, as it affects the distinctive ability of the transform. When σ is used to give a narrow window in time, then information is getting lost in frequency domain. In the opposite situation, information for our signal reconstruction is being lost in the time domain. In the following figures is explained exactly the above conclusion using the Gaussian window of length 128 and 1024. These values determine the frequencies in which the Fourier Transform is computed. The computation took place in Matlab® environment.

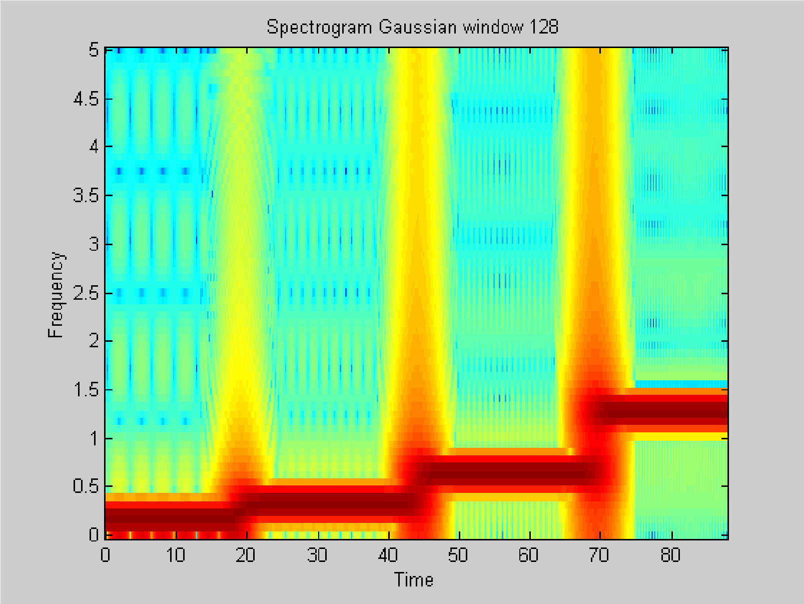


Fig.2. STFT using window parameter 128

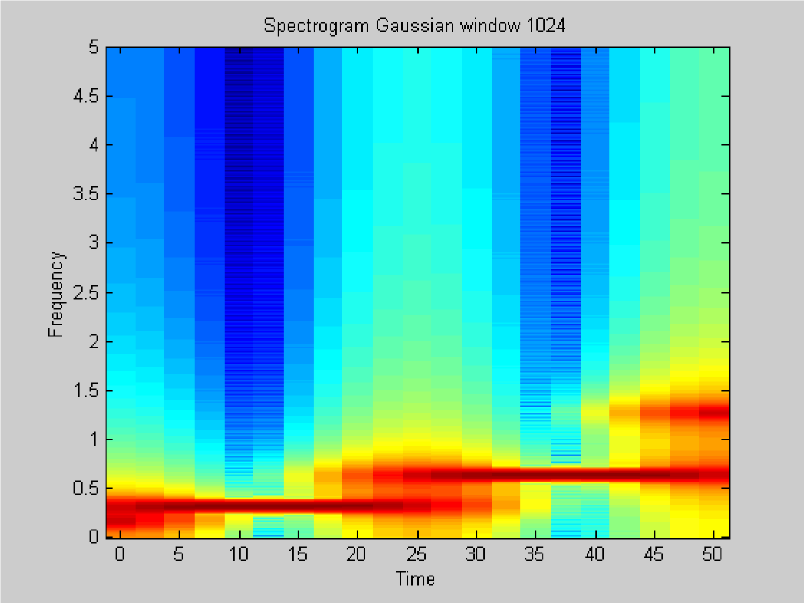


Fig.3. STFT using window parameter 1024

Furthermore, we had the same sine function's computation using other windows that prove to be quite effective in Space Time Frequency Transform. The next three figures show the resulting spectrograms with Hamming, Hanning and Kaiser window, for a window length value of 512. As can be noticed, the Hamming window has a low distinctive ability in the frequency domain. This leads to an image blurring, without letting us take much informaton about our signal in the frequency domain.

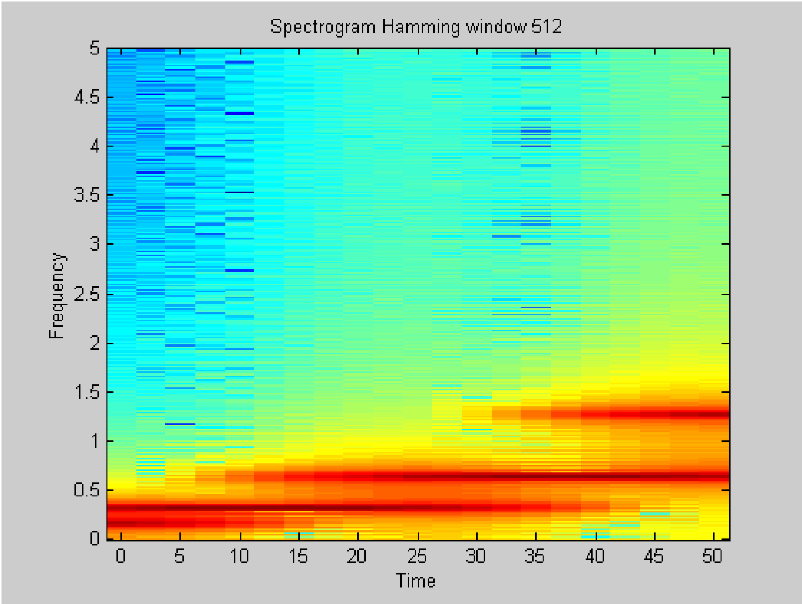


Fig.4. STFT using Hamming window

On the other hand, Hanning window is much more informative than the Hamming one. It gives great information in the frequency domain, using exactly the same parameters we used for the previous computation (window length 512). The following figure presents the image of our signal and, as can be easily seen, it provides the frequency values with great precision. The last spectrogram has been computed using Kaiser pulse, and is assumed to be of quite high distinctive ability.

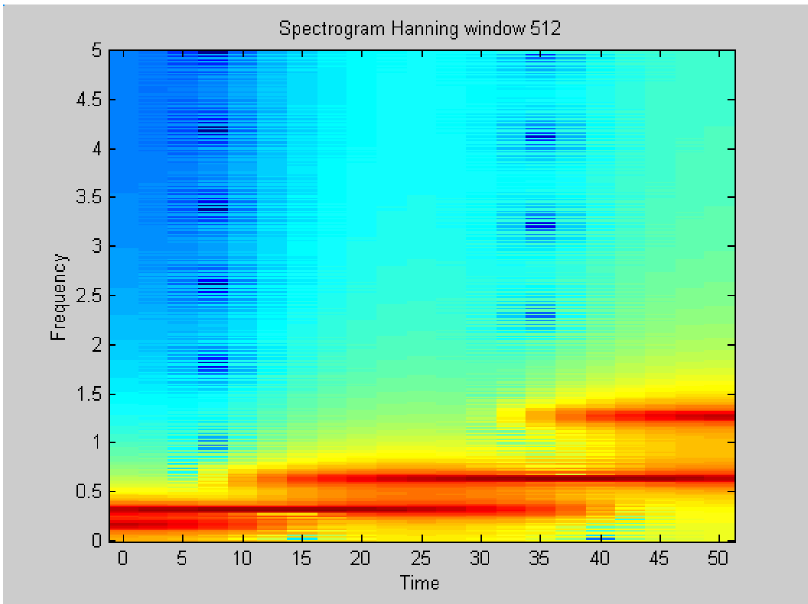


Fig.5. STFT using Hanning window

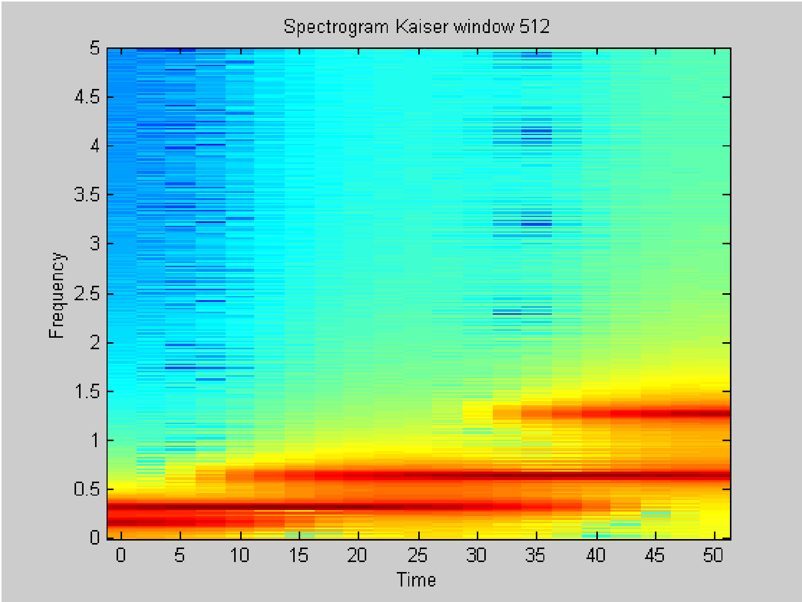


Fig.6. STFT using Kaiser window

To sum up, as far as STFT is concerned, we found out with the figures' help, that the four different sins of our signal are presented consequently in time, but whether we can have or not high distinctive ability, in time or frequency domain, depends on the window length and other parameters of the window functions, as well as the type of the window function we use.

We went on using the same signal in order to find out the differences between the mother wavelets in Continuous Wavelet Transform. We also tried to compare the two transforms, their attitude in time and frequency (or scale) domain. The following figures are the results of CWT computation (scalograms) using two different Gaussian wavelets (Gaussian1 and Gaussian8), Mexican Hat wavelet, Morlet wavelet (a complex sinusoid windowed with a Gaussian envelope) and Symlet wavelet. In the plots the x-axis represents position along the signal (time), the y-axis represents scale, and the color at each x-y point represents the magnitude of the wavelet coefficient C. The darker one point is, the smaller coefficient it represents. Recall that the higher scales correspond to the most "stretched" wavelets. The more stretched the wavelet, the longer the portion of the signal with which it is being compared, and thus the coarser the signal features being measured by the wavelet coefficients. What we tried to work out with the following figures was to build an image with the less scale computation, which means an informative result without comparing our signal with infinite shifted and dilated versions of the mother wavelet we used. This will lessen the computation time, and taking under consideration that our research has the goal to image radar targets, one can understand that computation time is priceless.

The process took place in Matlab® environment using 256 or 512 different scaled (stretched) versions of the wavelet. Gaussian1 wavelet, compared with Gaussian8, seems to be quite clear from the very first scales, whereas Gaussian8 gives information in greater scales. Of course, such a conclusion cannot be stable for every signal under processing. It depends on the signal form that is used each time.

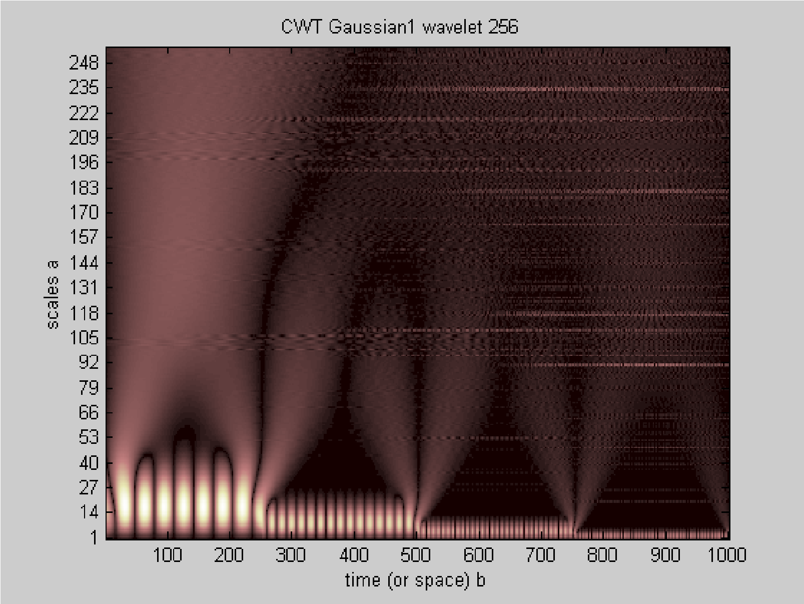


Fig.7. CWT using Gaussian1 wavelet

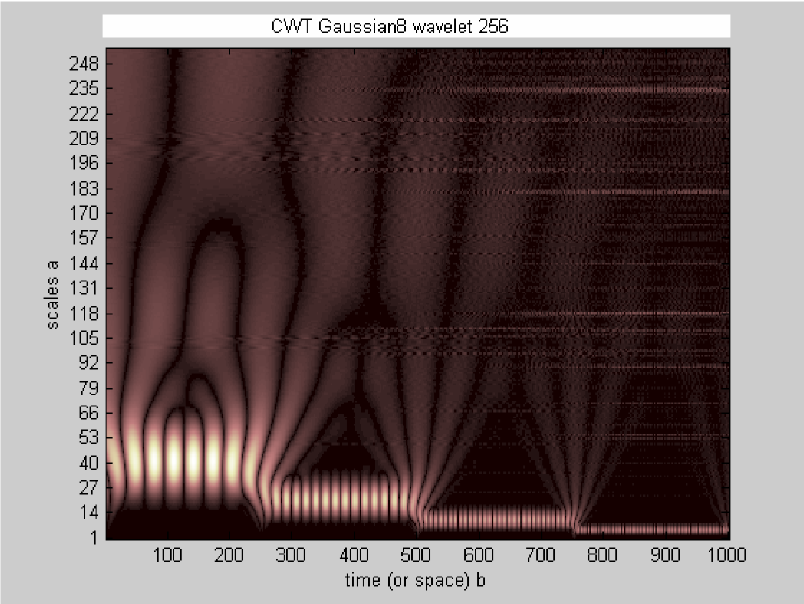


Fig.8. CWT using Gaussian8 wavelet

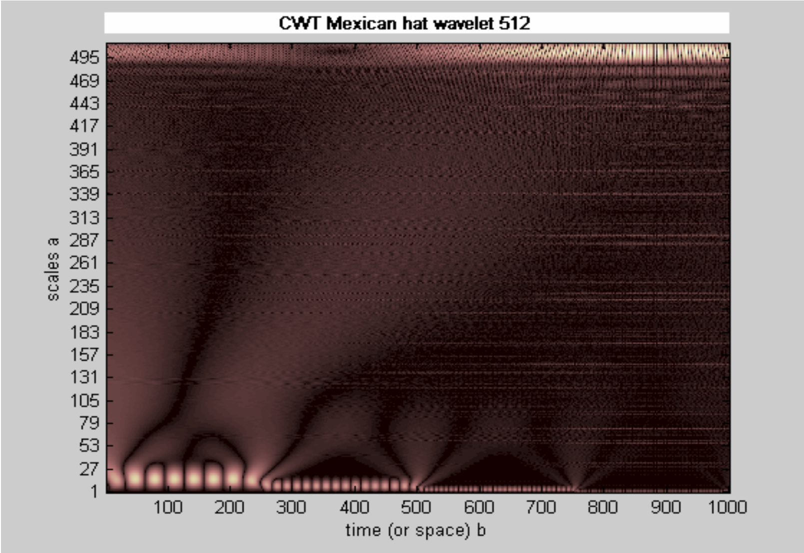


Fig.9. CWT using Mexican Hat wavelet

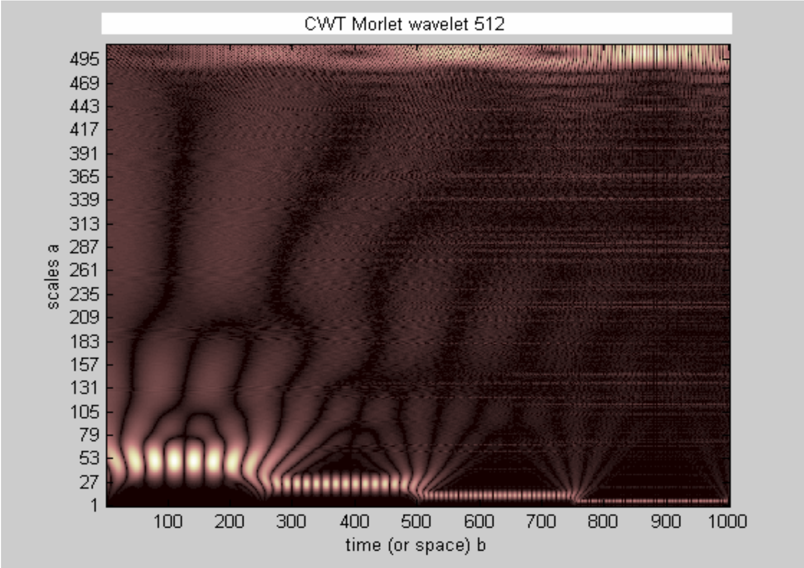


Fig.10. CWT using Morlet wavelet

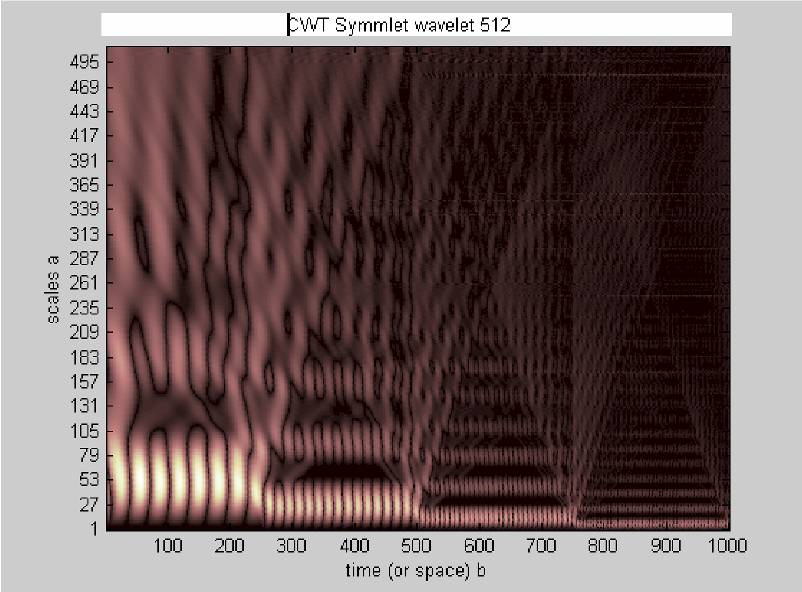


Fig.11. CWT using Symlet wavelet

In CWT, in low frequencies (large scales), the distinct ability in the time domain is low. Nevertheless, it does not appear to be a problem. While frequency is getting greater, which means smaller scales, we get better results in time domain. On the contrary, in scale domain the processing proves to be more effective.

As can be seen from the figures, wavelet analysis is capable of revealing aspects of data that other signal analysis techniques (like STFT) miss, aspects like breakdown points, discontinuities in higher derivatives, and self-similarity. Furthermore, because it affords a different view of data than those presented by traditional techniques, wavelet analysis can often compress or de-noise a signal without appreciable degradation.

Our research continued with the same processing but with the use of a more complex signal, a chirp pulse with quadratic instantaneous frequency deviation (1 KHz sample rate), that is presented below:

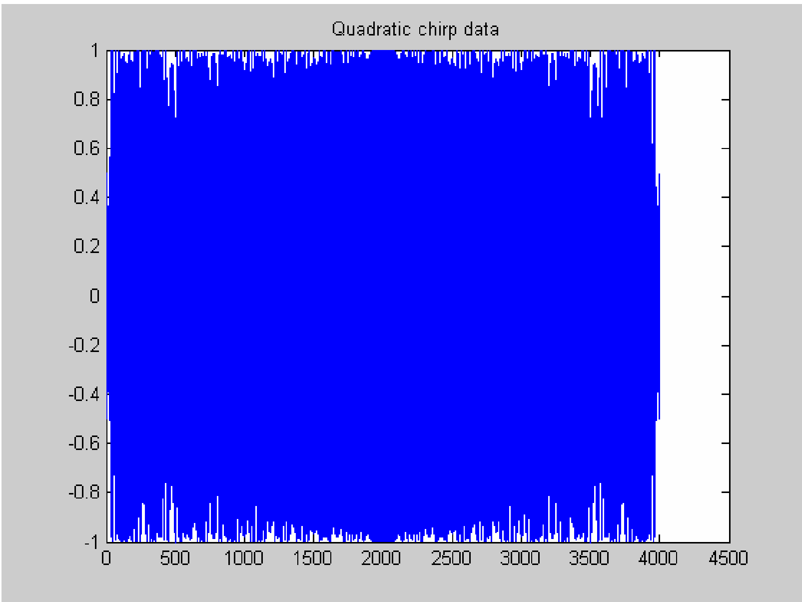


Fig.12. Quadratic chirp data

We used the same window functions to compute the spectrograms, as well as the same mother wavelets.

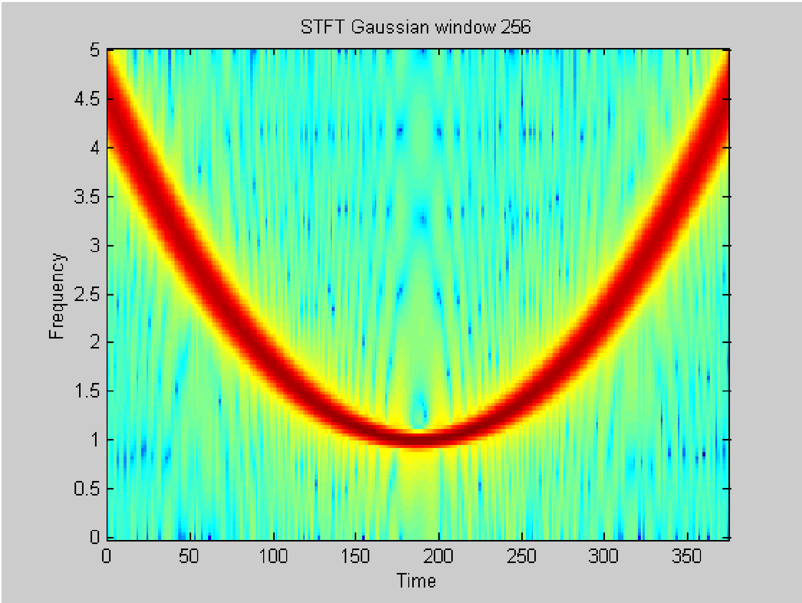


Fig.13. STFT using Gaussian window

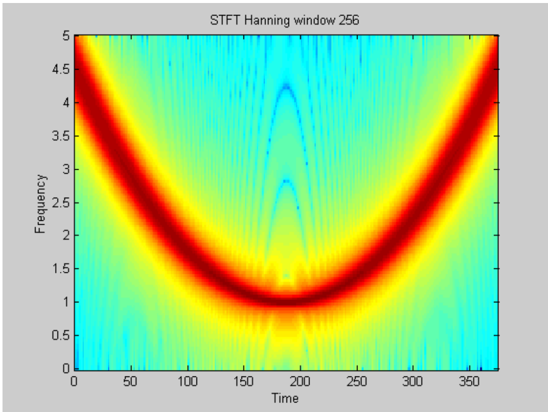


Fig.14. STFT using Hanning window

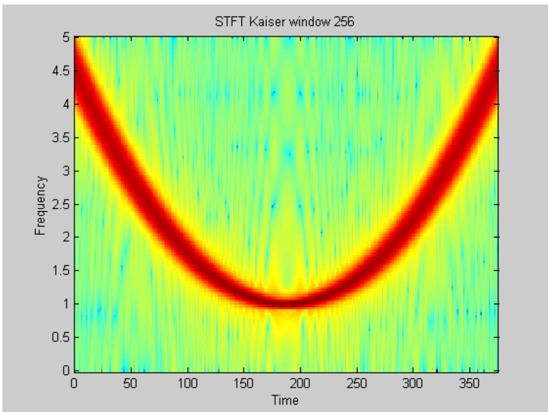


Fig.15. STFT using Kaiser window

The results and the conclusions of the second example were almost identical to the first example, which ensured that what we noticed in the previous figures was correct.

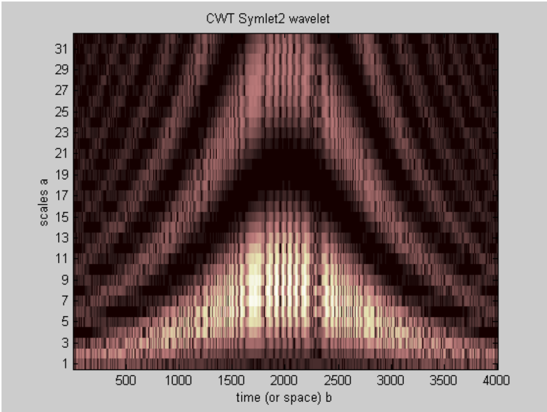


Fig.16. CWT using Symlet2 wavelet

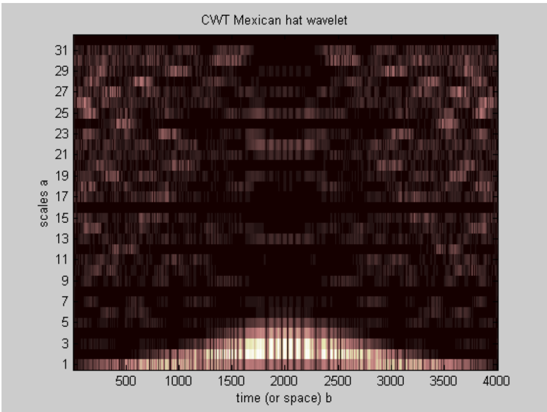


Fig.17. CWT using Mexican Hat wavelet

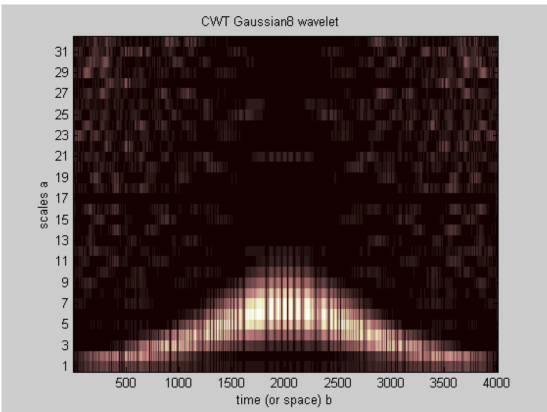


Fig.18. CWT using Gaussian8 wavelet

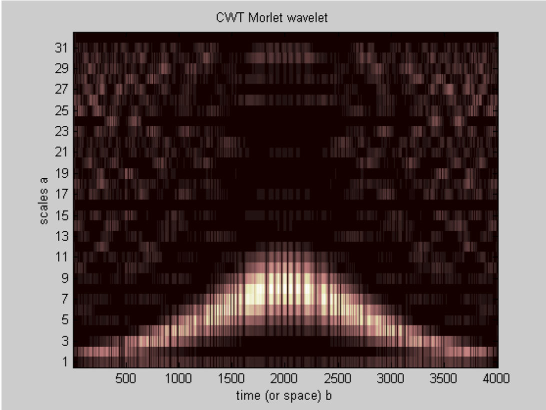


Fig.19. CWT using Morlet wavelet

6. IMAGE GENERATION FOR A SIMULATED TARGET

In this section, we formulated a model for the received scattered radar signal of a synthetic (operative) aircraft target. According to [7], a radar target can be considered as a collection of finite number of scatteri sum of scattered signals from each center. Based on this concept, and using the procedure described in [7], we worked on Matlab® environment and generated an aircraft target. We used nine (9) scattering centers that are presented in Figure 20, as well as 31 different look angles. The rotation center for our model is on coordinates [4530,0]. The radar is located on [0,0] and each of its bursts is consisted of 16 pulses (frequencies). Furthermore, we defined the scattering intensity for all scatterers and the Cartesian coordinates for all scatterers relative to the center of rotation. To improve even more our scattered signal before imaging the target, we interpolated the frequency-angular domain data to Cartesian coordinates. The image of the target acquired, after all the described procedure and the final Continuous Wavelet Transform, using a Symlet wavelet, is shown in Figure 21. The image is quite clear and the plot is intense on the scatterers' position, forming in that way our target.

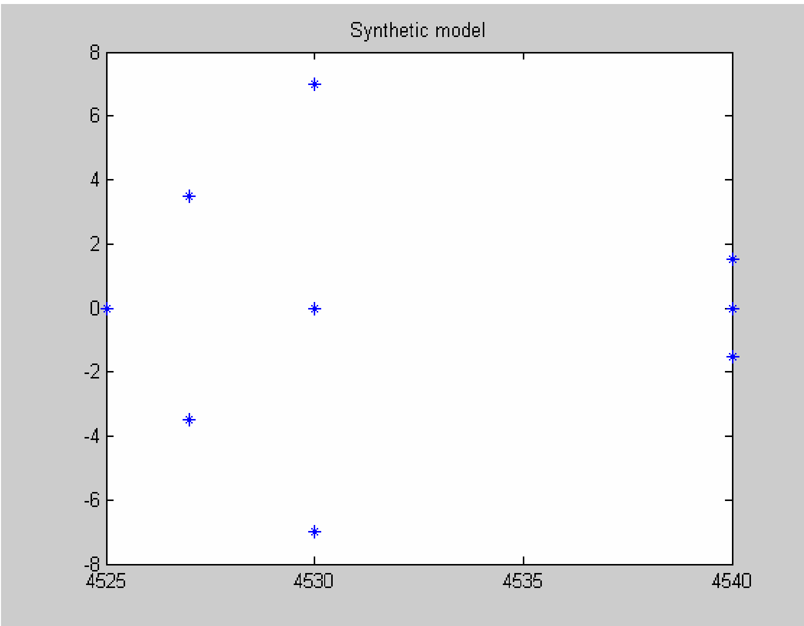


Fig.20. Scatterers of the Synthetic model

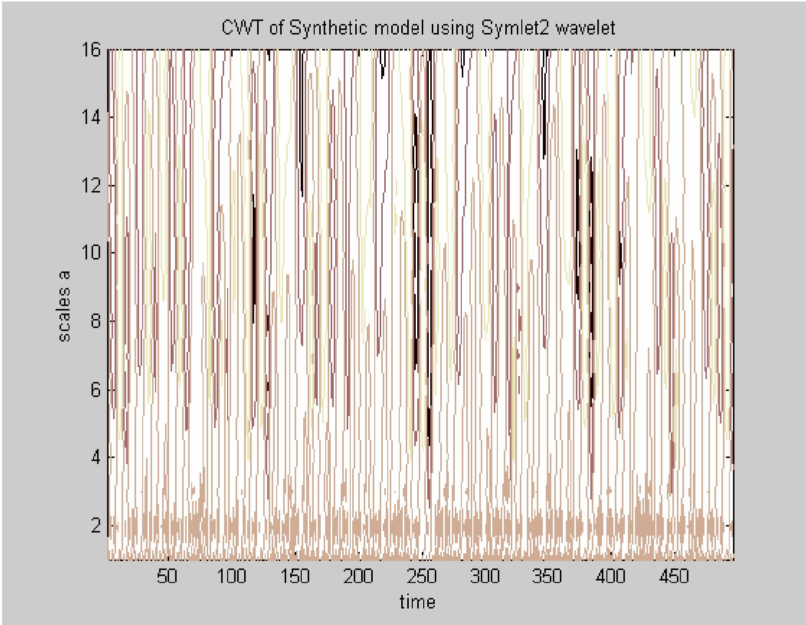


Fig.21. CWT of the Synthetic model

7. CONCLUSION

We have seen that both the Short Time Fourier Tranform and the Continuous Wavelet Transform represent ways to divide and examine more thoroughly the time-frequency (or time-scale) plane. We used some simple functions to show the differences, the benefits and drawbacks, of each method. Our research even concluded comparison between different window functions or mother wavelets. The two methods were demonstrated in detail, whereas CWT was also used to form the image of a simulated target.

The results, as far as the recognition accuracy of the target is concerned, were satisfying, showing that the proposed technique – Continous Wavelet Transform – has significant advantages in signal processing, and especially in radar target recognition, that is the goal of our research.

This means that the echoes of radar signals can be collected and processed to generate a two-dimensional representation of a target. Nevertheless, a simple time-frequency transform is not enough, when a non-cooperative target is being studied, due to its radial and rotational motion that leads to a blurred image [8]. This issue is extremely important and is an area of future research.

REFERENCES

[1] L.Durak, O.Arikan, "Short-Time Fourier Transform : Two Fundamental Properties and an Optimal Implementation", IEEE Trans. SP, Vol.51(5), May 2003, pp. 1231-1242

[2] O.Rioul, M.Vetterli, "Wavelets and Signal Processing", IEEE SP Magazine, Vol.8, Oct. 1991, pp.14-38

[3] V.Chen, H.Ling, "Time-Frequency Transforms for Radar Imaging and Signal Analysis", Artech House, Boston-London, 2002

[4] P.Goupillaud, A.Grossmann, J.Morlet, "Cycle-Octave and Related Transforms in Seismic Signal Analysis", Geoexploration, Vol.23, 1984, pp.85-102

[5] I.Daubechies, "The Wavelet Transform, Time-Frequency Localization and Signal Analysis", IEEE Trans. IT, Vol.36(5), Sept.1990, pp.961-1005

[6] J.Sadowsky, "Investigation of Signal Characteristics Using the Continuous Wavelet Transform", Johns Hopkins APL Technical Digest, Vol.17(3), 1996, pp.258-269

[7] J.Odendaal, E.Barnard, C.W.Pistorius, "Two-Dimensional Superresolution Radar Imaging Using the MUSIC Algorithm", IEEE Trans. Ant. and Propagation, Vol.42(10), Oct.1994, pp.1386-1391

[8] J.Son, G.Thomas, B. Flores, "Range-Doppler Radar Imaging and Motion Compensation", Artech House, Boston-London, 2001